

NAG Library Function Document

nag_ztgsja (f08ysc)

1 Purpose

nag_ztgsja (f08ysc) computes the generalized singular value decomposition (GSVD) of two complex upper trapezoidal matrices A and B , where A is an m by n matrix and B is a p by n matrix.

A and B are assumed to be in the form returned by nag_zggsvp (f08vsc).

2 Specification

```
#include <nag.h>
#include <nagf08.h>

void nag_ztgsja (Nag_OrderType order, Nag_ComputeUType jobu,
                Nag_ComputeVType jobv, Nag_ComputeQType jobq, Integer m, Integer p,
                Integer n, Integer k, Integer l, Complex a[], Integer pda, Complex b[],
                Integer pdb, double tola, double tolb, double alpha[], double beta[],
                Complex u[], Integer pdu, Complex v[], Integer pdv, Complex q[],
                Integer pdq, Integer *ncycle, NagError *fail)
```

3 Description

nag_ztgsja (f08ysc) computes the GSVD of the matrices A and B which are assumed to have the form as returned by nag_zggsvp (f08vsc)

$$A = \begin{cases} \begin{pmatrix} n-k-l & k & l \\ k & \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \end{pmatrix} \\ l & \begin{pmatrix} 0 & 0 & A_{23} \end{pmatrix} \\ m-k-l & \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \end{pmatrix}, & \text{if } m-k-l \geq 0; \\ \begin{pmatrix} n-k-l & k & l \\ k & \begin{pmatrix} 0 & A_{12} & A_{13} \\ 0 & 0 & A_{23} \end{pmatrix} \\ m-k & \begin{pmatrix} 0 & 0 & A_{23} \end{pmatrix} \end{pmatrix}, & \text{if } m-k-l < 0; \end{cases}$$

$$B = \begin{pmatrix} n-k-l & k & l \\ l & \begin{pmatrix} 0 & 0 & B_{13} \\ 0 & 0 & 0 \end{pmatrix} \\ p-l & \begin{pmatrix} 0 & 0 & 0 \end{pmatrix} \end{pmatrix},$$

where the k by k matrix A_{12} and the l by l matrix B_{13} are nonsingular upper triangular, A_{23} is l by l upper triangular if $m-k-l \geq 0$ and is $(m-k)$ by l upper trapezoidal otherwise.

nag_ztgsja (f08ysc) computes unitary matrices Q , U and V , diagonal matrices D_1 and D_2 , and an upper triangular matrix R such that

$$U^H A Q = D_1 \begin{pmatrix} 0 & R \\ & \end{pmatrix}, \quad V^H B Q = D_2 \begin{pmatrix} 0 & R \\ & \end{pmatrix}.$$

Optionally Q , U and V may or may not be computed, or they may be premultiplied by matrices Q_1 , U_1 and V_1 respectively.

If $(m - k - l) \geq 0$ then D_1 , D_2 and R have the form

$$D_1 = \begin{matrix} & k & l \\ & I & 0 \\ & 0 & C \\ m - k - l & 0 & 0 \end{matrix},$$

$$D_2 = \begin{matrix} & k & l \\ & 0 & S \\ p - l & 0 & 0 \end{matrix},$$

$$R = \begin{matrix} & k & l \\ k & R_{11} & R_{12} \\ l & 0 & R_{22} \end{matrix},$$

where $C = \text{diag}(\alpha_{k+1}, \dots, \alpha_{k+l})$, $S = \text{diag}(\beta_{k+1}, \dots, \beta_{k+l})$.

If $(m - k - l) < 0$ then D_1 , D_2 and R have the form

$$D_1 = \begin{matrix} & k & m - k & k + l - m \\ & I & 0 & 0 \\ m - k & 0 & C & 0 \end{matrix},$$

$$D_2 = \begin{matrix} & k & m - k & k + l - m \\ m - k & 0 & S & 0 \\ k + l - m & 0 & 0 & I \\ p - l & 0 & 0 & 0 \end{matrix},$$

$$R = \begin{matrix} & k & m - k & k + l - m \\ & R_{11} & R_{12} & R_{13} \\ m - k & 0 & R_{22} & R_{23} \\ k + l - m & 0 & 0 & R_{33} \end{matrix},$$

where $C = \text{diag}(\alpha_{k+1}, \dots, \alpha_m)$, $S = \text{diag}(\beta_{k+1}, \dots, \beta_m)$.

In both cases the diagonal matrix C has real non-negative diagonal elements, the diagonal matrix S has real positive diagonal elements, so that S is nonsingular, and $C^2 + S^2 = 1$. See Section 2.3.5.3 of Anderson *et al.* (1999) for further information.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **order** – Nag_OrderType

Input

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

- 2: **jobu** – Nag_ComputeUType Input
On entry: if **jobu** = Nag_AllU, **u** must contain a unitary matrix U_1 on entry, and the product U_1U is returned.
 If **jobu** = Nag_InitU, **u** is initialized to the unit matrix, and the unitary matrix U is returned.
 If **jobu** = Nag_NotU, U is not computed.
Constraint: **jobu** = Nag_InitU, Nag_AllU or Nag_NotU.
- 3: **jobv** – Nag_ComputeVType Input
On entry: if **jobv** = Nag_ComputeV, **v** must contain a unitary matrix V_1 on entry, and the product V_1V is returned.
 If **jobv** = Nag_InitV, **v** is initialized to the unit matrix, and the unitary matrix V is returned.
 If **jobv** = Nag_NotV, V is not computed.
Constraint: **jobv** = Nag_ComputeV, Nag_InitV or Nag_NotV.
- 4: **jobq** – Nag_ComputeQType Input
On entry: if **jobq** = Nag_ComputeQ, **q** must contain a unitary matrix Q_1 on entry, and the product Q_1Q is returned.
 If **jobq** = Nag_InitQ, **q** is initialized to the unit matrix, and the unitary matrix Q is returned.
 If **jobq** = Nag_NotQ, Q is not computed.
Constraint: **jobq** = Nag_ComputeQ, Nag_InitQ or Nag_NotQ.
- 5: **m** – Integer Input
On entry: m , the number of rows of the matrix A .
Constraint: $m \geq 0$.
- 6: **p** – Integer Input
On entry: p , the number of rows of the matrix B .
Constraint: $p \geq 0$.
- 7: **n** – Integer Input
On entry: n , the number of columns of the matrices A and B .
Constraint: $n \geq 0$.
- 8: **k** – Integer Input
 9: **l** – Integer Input
On entry: **k** and **l** specify the sizes, k and l , of the subblocks of A and B , whose GSVD is to be computed by nag_ztgsja (f08ysc).
- 10: **a**[*dim*] – Complex Input/Output
Note: the dimension, *dim*, of the array **a** must be at least
 $\max(1, \mathbf{pda} \times \mathbf{n})$ when **order** = Nag_ColMajor;
 $\max(1, \mathbf{m} \times \mathbf{pda})$ when **order** = Nag_RowMajor.
 Where $\mathbf{A}(i, j)$ appears in this document, it refers to the array element
 $\mathbf{a}[(j - 1) \times \mathbf{pda} + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{a}[(i - 1) \times \mathbf{pda} + j - 1]$ when **order** = Nag_RowMajor.
On entry: the m by n matrix A .

On exit: if $m - k - l \geq 0$, $\mathbf{A}(1 : k + l, n - k - l + 1 : n)$ contains the $(k + l)$ by $(k + l)$ upper triangular matrix R .

If $m - k - l < 0$, $\mathbf{A}(1 : m, n - k - l + 1 : n)$ contains the first m rows of the $(k + l)$ by $(k + l)$ upper triangular matrix R , and the submatrix R_{33} is returned in $\mathbf{B}(m - k + 1 : l, n + m - k - l + 1 : n)$.

11: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraints:

if **order** = Nag_ColMajor, **pda** \geq max(1, **m**);
if **order** = Nag_RowMajor, **pda** \geq max(1, **n**).

12: **b[*dim*]** – Complex *Input/Output*

Note: the dimension, *dim*, of the array **b** must be at least

max(1, **pdb** \times **n**) when **order** = Nag_ColMajor;
max(1, **p** \times **pdb**) when **order** = Nag_RowMajor.

Where $\mathbf{B}(i, j)$ appears in this document, it refers to the array element

$\mathbf{b}[(j - 1) \times \mathbf{pdb} + i - 1]$ when **order** = Nag_ColMajor;
 $\mathbf{b}[(i - 1) \times \mathbf{pdb} + j - 1]$ when **order** = Nag_RowMajor.

On entry: the p by n matrix B .

On exit: if $m - k - l < 0$, $\mathbf{B}(m - k + 1 : l, n + m - k - l + 1 : n)$ contains the submatrix R_{33} of R .

13: **pdb** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = Nag_ColMajor, **pdb** \geq max(1, **p**);
if **order** = Nag_RowMajor, **pdb** \geq max(1, **n**).

14: **tola** – double *Input*

15: **tolb** – double *Input*

On entry: **tola** and **tolb** are the convergence criteria for the Jacobi–Kogbetliantz iteration procedure. Generally, they should be the same as used in the preprocessing step performed by nag_zggsvp (f08vsc), say

$$\begin{aligned} \mathbf{tola} &= \max(\mathbf{m}, \mathbf{n}) \|A\| \epsilon, \\ \mathbf{tolb} &= \max(\mathbf{p}, \mathbf{n}) \|B\| \epsilon, \end{aligned}$$

where ϵ is the *machine precision*.

16: **alpha[n]** – double *Output*

On exit: see the description of **beta**.

17: **beta[n]** – double *Output*

On exit: **alpha** and **beta** contain the generalized singular value pairs of A and B ;

alpha[i] = 1, **beta**[i] = 0, for $i = 0, 1, \dots, k - 1$, and

if $m - k - l \geq 0$, **alpha**[i] = α_i , **beta**[i] = β_i , for $i = k, \dots, k + l - 1$, or

if $m - k - l < 0$, $\mathbf{alpha}[i] = \alpha_i$, $\mathbf{beta}[i] = \beta_i$, for $i = k, \dots, m - 1$ and $\mathbf{alpha}[i] = 0$, $\mathbf{beta}[i] = 1$, for $i = m, \dots, k + l - 1$.

Furthermore, if $k + l < n$, $\mathbf{alpha}[i] = \mathbf{beta}[i] = 0$, for $i = k + l, \dots, n - 1$.

18: $\mathbf{u}[dim]$ – Complex

Input/Output

Note: the dimension, dim , of the array \mathbf{u} must be at least

$\max(1, \mathbf{pdu} \times \mathbf{m})$ when $\mathbf{jobu} = \text{Nag_AllU}$ or Nag_InitU and $\mathbf{order} = \text{Nag_ColMajor}$;
 $\max(1, \times \mathbf{pdu})$ when $\mathbf{jobu} = \text{Nag_AllU}$ or Nag_InitU and $\mathbf{order} = \text{Nag_RowMajor}$;
 1 otherwise.

The (i, j) th element of the matrix U is stored in

$\mathbf{u}[(j - 1) \times \mathbf{pdu} + i - 1]$ when $\mathbf{order} = \text{Nag_ColMajor}$;
 $\mathbf{u}[(i - 1) \times \mathbf{pdu} + j - 1]$ when $\mathbf{order} = \text{Nag_RowMajor}$.

On entry: if $\mathbf{jobu} = \text{Nag_AllU}$, \mathbf{u} must contain an m by m matrix U_1 (usually the unitary matrix returned by `nag_zggsvp` (`f08vsc`)).

On exit: if $\mathbf{jobu} = \text{Nag_InitU}$, \mathbf{u} contains the unitary matrix U .

If $\mathbf{jobu} = \text{Nag_AllU}$, \mathbf{u} contains the product $U_1 U$.

If $\mathbf{jobu} = \text{Nag_NotU}$, \mathbf{u} is not referenced.

19: \mathbf{pdu} – Integer

Input

On entry: the stride separating row or column elements (depending on the value of \mathbf{order}) in the array \mathbf{u} .

Constraints:

if $\mathbf{order} = \text{Nag_ColMajor}$,
 if $\mathbf{jobu} \neq \text{Nag_NotU}$, $\mathbf{pdu} \geq \max(1, \mathbf{m})$;
 otherwise $\mathbf{pdu} \geq 1$.;
 if $\mathbf{order} = \text{Nag_RowMajor}$,
 if $\mathbf{jobu} = \text{Nag_AllU}$ or Nag_InitU , $\mathbf{pdu} \geq \max(1, \mathbf{m})$;
 otherwise $\mathbf{pdu} \geq 1$.

20: $\mathbf{v}[dim]$ – Complex

Input/Output

Note: the dimension, dim , of the array \mathbf{v} must be at least

$\max(1, \mathbf{pdv} \times \mathbf{p})$ when $\mathbf{jobv} = \text{Nag_ComputeV}$ or Nag_InitV and $\mathbf{order} = \text{Nag_ColMajor}$;
 $\max(1, \times \mathbf{pdv})$ when $\mathbf{jobv} = \text{Nag_ComputeV}$ or Nag_InitV and $\mathbf{order} = \text{Nag_RowMajor}$;
 1 otherwise.

The (i, j) th element of the matrix V is stored in

$\mathbf{v}[(j - 1) \times \mathbf{pdv} + i - 1]$ when $\mathbf{order} = \text{Nag_ColMajor}$;
 $\mathbf{v}[(i - 1) \times \mathbf{pdv} + j - 1]$ when $\mathbf{order} = \text{Nag_RowMajor}$.

On entry: if $\mathbf{jobv} = \text{Nag_ComputeV}$, \mathbf{v} must contain an p by p matrix V_1 (usually the unitary matrix returned by `nag_zggsvp` (`f08vsc`)).

On exit: if $\mathbf{jobv} = \text{Nag_InitV}$, \mathbf{v} contains the unitary matrix V .

If $\mathbf{jobv} = \text{Nag_ComputeV}$, \mathbf{v} contains the product $V_1 V$.

If $\mathbf{jobv} = \text{Nag_NotV}$, \mathbf{v} is not referenced.

21: \mathbf{pdv} – Integer

Input

On entry: the stride separating row or column elements (depending on the value of \mathbf{order}) in the array \mathbf{v} .

Constraints:

if **order** = Nag_ColMajor,
 if **jobv** \neq Nag_NotV, **pdv** \geq max(1, **p**);
 otherwise **pdv** \geq 1.;
 if **order** = Nag_RowMajor,
 if **jobv** = Nag_ComputeV or Nag_InitV, **pdv** \geq max(1, **p**);
 otherwise **pdv** \geq 1..

22: **q**[*dim*] – Complex

Input/Output

Note: the dimension, *dim*, of the array **q** must be at least

max(1, **pdq** \times **n**) when **jobq** = Nag_ComputeQ or Nag_InitQ and **order** = Nag_ColMajor;
 max(1, \times **pdq**) when **jobq** = Nag_ComputeQ or Nag_InitQ and **order** = Nag_RowMajor;
 1 otherwise.

The (*i*, *j*)th element of the matrix *Q* is stored in

q[(*j* – 1) \times **pdq** + *i* – 1] when **order** = Nag_ColMajor;
q[(*i* – 1) \times **pdq** + *j* – 1] when **order** = Nag_RowMajor.

On entry: if **jobq** = Nag_ComputeQ, **q** must contain an *n* by *n* matrix Q_1 (usually the unitary matrix returned by nag_zggsvp (f08vsc)).

On exit: if **jobq** = Nag_InitQ, **q** contains the unitary matrix *Q*.

If **jobq** = Nag_ComputeQ, **q** contains the product Q_1Q .

If **jobq** = Nag_NotQ, **q** is not referenced.

23: **pdq** – Integer

Input

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **q**.

Constraints:

if **order** = Nag_ColMajor,
 if **jobq** \neq Nag_NotQ, **pdq** \geq max(1, **n**);
 otherwise **pdq** \geq 1.;
 if **order** = Nag_RowMajor,
 if **jobq** = Nag_ComputeQ or Nag_InitQ, **pdq** \geq max(1, **n**);
 otherwise **pdq** \geq 1..

24: **ncycle** – Integer *

Output

On exit: the number of cycles required for convergence.

25: **fail** – NagError *

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument \langle *value* \rangle had an illegal value.

NE_CONVERGENCE

The procedure does not converge after 40 cycles.

NE_ENUM_INT_2

On entry, **jobq** = *⟨value⟩*, **pdq** = *⟨value⟩*, **n** = *⟨value⟩*.

Constraint: if **jobq** = Nag_ComputeQ or Nag_InitQ, **pdq** \geq max(1, **n**);
otherwise **pdq** \geq 1.

On entry, **jobu** = *⟨value⟩*, **pdu** = *⟨value⟩*, **m** = *⟨value⟩*.

Constraint: if **jobu** = Nag_AllU or Nag_InitU, **pdu** \geq max(1, **m**);
otherwise **pdu** \geq 1.

On entry, **jobu** = *⟨value⟩*, **pdu** = *⟨value⟩* and **m** = *⟨value⟩*.

Constraint: if **jobu** \neq Nag_NotU, **pdu** \geq max(1, **m**);
otherwise **pdu** \geq 1.

On entry, **jobv** = *⟨value⟩*, **pdv** = *⟨value⟩*, **p** = *⟨value⟩*.

Constraint: if **jobv** = Nag_ComputeV or Nag_InitV, **pdv** \geq max(1, **p**);
otherwise **pdv** \geq 1.

On entry, **jobv** = *⟨value⟩*, **pdv** = *⟨value⟩* and **p** = *⟨value⟩*.

Constraint: if **jobv** \neq Nag_NotV, **pdv** \geq max(1, **p**);
otherwise **pdv** \geq 1.

On entry, **pdq** = *⟨value⟩*, **jobq** = *⟨value⟩* and **n** = *⟨value⟩*.

Constraint: if **jobq** \neq Nag_NotQ, **pdq** \geq max(1, **n**);
otherwise **pdq** \geq 1.

NE_INT

On entry, **m** = *⟨value⟩*.

Constraint: **m** \geq 0.

On entry, **n** = *⟨value⟩*.

Constraint: **n** \geq 0.

On entry, **p** = *⟨value⟩*.

Constraint: **p** \geq 0.

On entry, **pda** = *⟨value⟩*.

Constraint: **pda** $>$ 0.

On entry, **pdb** = *⟨value⟩*.

Constraint: **pdb** $>$ 0.

On entry, **pdq** = *⟨value⟩*.

Constraint: **pdq** $>$ 0.

On entry, **pdu** = *⟨value⟩*.

Constraint: **pdu** $>$ 0.

On entry, **pdv** = *⟨value⟩*.

Constraint: **pdv** $>$ 0.

NE_INT_2

On entry, **pda** = *⟨value⟩* and **m** = *⟨value⟩*.

Constraint: **pda** \geq max(1, **m**).

On entry, **pda** = *⟨value⟩* and **n** = *⟨value⟩*.

Constraint: **pda** \geq max(1, **n**).

On entry, **pdb** = *⟨value⟩* and **n** = *⟨value⟩*.

Constraint: **pdb** \geq max(1, **n**).

On entry, **pdb** = $\langle value \rangle$ and **p** = $\langle value \rangle$.
 Constraint: **pdb** $\geq \max(1, \mathbf{p})$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

The computed generalized singular value decomposition is nearly the exact generalized singular value decomposition for nearby matrices $(A + E)$ and $(B + F)$, where

$$\|E\|_2 = O\epsilon\|A\|_2 \quad \text{and} \quad \|F\|_2 = O\epsilon\|B\|_2,$$

and ϵ is the *machine precision*. See Section 4.12 of Anderson *et al.* (1999) for further details.

8 Parallelism and Performance

nag_ztgsja (f08ysc) is not threaded by NAG in any implementation.

nag_ztgsja (f08ysc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The real analogue of this function is nag_dtgsja (f08yec).

10 Example

This example finds the generalized singular value decomposition

$$A = U\Sigma_1(0 \ R)Q^H, \quad B = V\Sigma_2(0 \ R)Q^H,$$

of the matrix pair (A, B) , where

$$A = \begin{pmatrix} 0.96 - 0.81i & -0.03 + 0.96i & -0.91 + 2.06i & -0.05 + 0.41i \\ -0.98 + 1.98i & -1.20 + 0.19i & -0.66 + 0.42i & -0.81 + 0.56i \\ 0.62 - 0.46i & 1.01 + 0.02i & 0.63 - 0.17i & -1.11 + 0.60i \\ 0.37 + 0.38i & 0.19 - 0.54i & -0.98 - 0.36i & 0.22 - 0.20i \\ 0.83 + 0.51i & 0.20 + 0.01i & -0.17 - 0.46i & 1.47 + 1.59i \\ 1.08 - 0.28i & 0.20 - 0.12i & -0.07 + 1.23i & 0.26 + 0.26i \end{pmatrix}$$

and

$$B = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_ztgsja (f08ysc) Example Program.
 *
 * Copyright 2011 Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
```



```

#include <nagf08.h>
#include <nagf16.h>
#include <nagx02.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    double          eps, norma, normb, tola, tolb;
    Integer         i, irank, j, k, l, m, n, ncycle, p, pda, pdb, pdu, pdv;
    Integer         pdq, printq, printr, printu, printv, vsize;
    Integer         exit_status = 0;

    /* Arrays */
    Complex         *a = 0, *b = 0, *q = 0, *u = 0, *v = 0;
    double          *alpha = 0, *beta = 0;
    char            nag_enum_arg[40];

    /* Nag Types */
    NagError        fail;
    Nag_OrderType   order;
    Nag_ComputeUType jobu;
    Nag_ComputeVType jobv;
    Nag_ComputeQType jobq;
    Nag_MatrixType  genmat = Nag_GeneralMatrix, upmat = Nag_UpperMatrix;
    Nag_DiagType    diag    = Nag_NonUnitDiag;
    Nag_LabelType   intlab = Nag_IntegerLabels;
    Nag_ComplexFormType brac  = Nag_BracketForm;

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I - 1]
#define B(I, J) b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J - 1]
#define B(I, J) b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif

    INIT_FAIL(fail);

    printf("nag_ztgsja (f08ysc) Example Program Results\n\n");

    /* Skip heading in data file */
    scanf("%*[\n]");
    scanf("%ld%ld%ld%*[\n]", &m, &n, &p);
    if (m < 0 || n < 0 || p < 0)
    {
        printf("Invalid m, n or p\n");
        exit_status = 1;
        goto END;
    }
    scanf(" %39s%*[\n]", nag_enum_arg);
    /* nag_enum_name_to_value (x04nac).
     * Converts NAG enum member name to value
     */
    jobu = (Nag_ComputeUType) nag_enum_name_to_value(nag_enum_arg);
    scanf(" %39s%*[\n]", nag_enum_arg);
    jobv = (Nag_ComputeVType) nag_enum_name_to_value(nag_enum_arg);
    scanf(" %39s%*[\n]", nag_enum_arg);
    jobq = (Nag_ComputeQType) nag_enum_name_to_value(nag_enum_arg);

    pdu = (jobu!=Nag_NotU?m:1);
    pdv = (jobv!=Nag_NotV?p:1);
    pdq = (jobq!=Nag_NotQ?n:1);
    vsize = (jobv!=Nag_NotV?p*m:1);
#ifdef NAG_COLUMN_MAJOR
    pda = m;
    pdb = p;
#else
    pda = n;

```

```

    pdb = n;
#endif

/* Read in Os or ls to determine whether matrices U, V, Q or R are to be
 * printed.
 */
scanf("%ld%ld%ld%ld%*[\n]",
      &printu, &printv, &printq, &printr);

/* Allocate memory */
if (!(a      = NAG_ALLOC(m*n, Complex)) ||
    !(b      = NAG_ALLOC(p*n, Complex)) ||
    !(alpha  = NAG_ALLOC(n, double)) ||
    !(beta   = NAG_ALLOC(n, double)) ||
    !(q      = NAG_ALLOC(pdq*pdq, Complex)) ||
    !(u      = NAG_ALLOC(pdu*pdu, Complex)) ||
    !(v      = NAG_ALLOC(vsize, Complex)))
{
    printf("Allocation failure\n");
    exit_status = -1;
    goto END;
}

/* Read the m by n matrix A and p by n matrix B from data file */
for (i = 1; i <= m; ++i)
    for (j = 1; j <= n; ++j)
        scanf(" ( %lf , %lf )", &A(i, j).re, &A(i, j).im);
scanf("%*[\n]");
for (i = 1; i <= p; ++i)
    for (j = 1; j <= n; ++j)
        scanf(" ( %lf , %lf )", &B(i, j).re, &B(i, j).im);
scanf("%*[\n]");

/* Compute tola and tolb as */
/* tola = max(m,n)*norm(A)*machepts */
/* tolb = max(p,n)*norm(B)*machepts */
nag_zge_norm(order, Nag_OneNorm, m, n, a, pda, &norma, &fail);
nag_zge_norm(order, Nag_OneNorm, p, n, b, pdb, &normb, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zge_norm (f16uac).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Compute tola and tolb using nag_machine_precision (x02ajc) */
eps = nag_machine_precision;
tola = MAX(m, n) * norma * eps;
tolb = MAX(p, n) * normb * eps;

/* Preprocess step:
 * compute transformations to reduce (A, B) to upper triangular form
 * (A = U1*S*(Q1^H), B = V1*T*(Q1^H))
 * using nag_zggsvp (f08vsc).
 */
nag_zggsvp(order, jobu, jobv, jobq, m, p, n, a, pda, b, pdb, tola, tolb, &k,
           &l, u, pdu, v, pdv, q, pdq, &fail);
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_zggsvp (f08vsc).\n%s\n", fail.message);
    exit_status = 1;
    goto END;
}

/* Compute the generalized singular value decomposition of preprocessed (A, B)
 * (A = U*D1*(O R)*(Q**H), B = V*D2*(O R)*(Q**H))
 * using nag_ztgsja (f08ysc).
 */
nag_ztgsja(order, jobu, jobv, jobq, m, p, n, k, l, a, pda, b, pdb, tola,
           tolb, alpha, beta, u, pdu, v, pdv, q, pdq, &ncycle, &fail);
if (fail.code != NE_NOERROR)

```

```

    {
        printf("Error from nag_ztgsja (f08ysc).\n%s\n", fail.message);
        exit_status = 1;
        goto END;
    }

/* Print the generalized singular value pairs alpha, beta */
irank = MIN(k + 1,m);
printf("Number of infinite generalized singular values (k): %5ld\n", k);
printf("Number of finite generalized singular values (l): %5ld\n", l);
printf("Effective Numerical rank of (A^H B^HT)^H (k+1): %5ld\n",
        irank);
printf("\nFinite generalized singular values:\n");

for (j = k; j < irank; ++j) printf("%45s%12.4e\n", "", alpha[j]/beta[j]);

printf("\nNumber of cycles of the Kogbetliantz method: %12ld\n\n", ncycle);

if (printu && jobu!=Nag_NotU) {
    fflush(stdout);
    nag_gen_complx_mat_print_comp(order, genmat, diag, m, m, u, pdu, brac,
        "%13.4e", "Orthogonal matrix U", intlab,
        NULL, intlab, NULL, 80, 0, NULL, &fail);
    if (fail.code != NE_NOERROR) goto PRINTERR;
}
if (printv && jobv!=Nag_NotV) {
    printf("\n");
    fflush(stdout);
    nag_gen_complx_mat_print_comp(order, genmat, diag, p, p, v, pdv, brac,
        "%13.4e", "Orthogonal matrix V", intlab,
        NULL, intlab, NULL, 80, 0, NULL, &fail);
    if (fail.code != NE_NOERROR) goto PRINTERR;
}
if (printq && jobq!=Nag_NotQ) {
    printf("\n");
    fflush(stdout);
    nag_gen_complx_mat_print_comp(order, genmat, diag, n, n, q, pdq, brac,
        "%13.4e", "Orthogonal matrix Q", intlab,
        NULL, intlab, NULL, 80, 0, NULL, &fail);
    if (fail.code != NE_NOERROR) goto PRINTERR;
}
if (printr) {
    printf("\n");
    fflush(stdout);
    nag_gen_complx_mat_print_comp(order, upmat, diag, irank, irank,
        &A(1, n - irank + 1), pda, brac, "%13.4e",
        "Non singular upper triangular matrix R",
        intlab, NULL, intlab, NULL, 80, 0, NULL,
        &fail);
}
PRINTERR:
if (fail.code != NE_NOERROR)
{
    printf("Error from nag_gen_real_mat_print_comp (x04cbc).\n%s\n",
        fail.message);
    exit_status = 1;
}

END:
NAG_FREE(a);
NAG_FREE(b);
NAG_FREE(alpha);
NAG_FREE(beta);
NAG_FREE(q);
NAG_FREE(u);
NAG_FREE(v);

return exit_status;
}

```

10.2 Program Data

nag_ztgsja (f08ysc) Example Program Data

```

6           4           2           : m, n and p

Nag_AllU           : jobu
Nag_ComputeV       : jobv
Nag_ComputeQ       : jobq

0           0           0           0           : print u, v, q, r?

( 0.96,-0.81) (-0.03, 0.96) (-0.91, 2.06) (-0.05, 0.41)
(-0.98, 1.98) (-1.20, 0.19) (-0.66, 0.42) (-0.81, 0.56)
( 0.62,-0.46) ( 1.01, 0.02) ( 0.63,-0.17) (-1.11, 0.60)
( 0.37, 0.38) ( 0.19,-0.54) (-0.98,-0.36) ( 0.22,-0.20)
( 0.83, 0.51) ( 0.20, 0.01) (-0.17,-0.46) ( 1.47, 1.59)
( 1.08,-0.28) ( 0.20,-0.12) (-0.07, 1.23) ( 0.26, 0.26) : matrix A

( 1.00, 0.00) ( 0.00, 0.00) (-1.00, 0.00) ( 0.00, 0.00)
( 0.00, 0.00) ( 1.00, 0.00) ( 0.00, 0.00) (-1.00, 0.00) : matrix B

```

10.3 Program Results

nag_ztgsja (f08ysc) Example Program Results

```

Number of infinite generalized singular values (k):      2
Number of finite generalized singular values (l):        2
Effective Numerical rank of (AH BHT)H (k+l):        4

```

Finite generalized singular values:

```

2.0720e+00
1.1058e+00

```

```

Number of cycles of the Kogbetliantz method:            2

```
