

## NAG Library Function Document

### nag\_real\_sym\_posdef\_tridiag\_lin\_solve (f04bgc)

#### 1 Purpose

nag\_real\_sym\_posdef\_tridiag\_lin\_solve (f04bgc) computes the solution to a real system of linear equations  $AX = B$ , where  $A$  is an  $n$  by  $n$  symmetric positive definite tridiagonal matrix and  $X$  and  $B$  are  $n$  by  $r$  matrices. An estimate of the condition number of  $A$  and an error bound for the computed solution are also returned.

#### 2 Specification

```
#include <nag.h>
#include <nagf04.h>

void nag_real_sym_posdef_tridiag_lin_solve (Nag_OrderType order, Integer n,
      Integer nrhs, double d[], double e[], double b[], Integer pdb,
      double *rcond, double *errbnd, NagError *fail)
```

#### 3 Description

$A$  is factorized as  $A = LDL^T$ , where  $L$  is a unit lower bidiagonal matrix and  $D$  is diagonal, and the factored form of  $A$  is then used to solve the system of equations.

#### 4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D (1999) *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia <http://www.netlib.org/lapack/lug>

Higham N J (2002) *Accuracy and Stability of Numerical Algorithms* (2nd Edition) SIAM, Philadelphia

#### 5 Arguments

- 1: **order** – Nag\_OrderType *Input*  
*On entry:* the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag\_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.  
*Constraint:* **order** = Nag\_RowMajor or Nag\_ColMajor.
- 2: **n** – Integer *Input*  
*On entry:* the number of linear equations  $n$ , i.e., the order of the matrix  $A$ .  
*Constraint:* **n**  $\geq$  0.
- 3: **nrhs** – Integer *Input*  
*On entry:* the number of right-hand sides  $r$ , i.e., the number of columns of the matrix  $B$ .  
*Constraint:* **nrhs**  $\geq$  0.
- 4: **d**[*dim*] – double *Input/Output*  
**Note:** the dimension, *dim*, of the array **d** must be at least  $\max(1, \mathbf{n})$ .

*On entry:* must contain the  $n$  diagonal elements of the tridiagonal matrix  $A$ .

*On exit:* if **fail.code** = NE\_NOERROR or NE\_RCOND, **d** is overwritten by the  $n$  diagonal elements of the diagonal matrix  $D$  from the  $LDL^T$  factorization of  $A$ .

5: **e**[*dim*] – double *Input/Output*

**Note:** the dimension, *dim*, of the array **e** must be at least  $\max(1, n - 1)$ .

*On entry:* must contain the  $(n - 1)$  subdiagonal elements of the tridiagonal matrix  $A$ .

*On exit:* if **fail.code** = NE\_NOERROR or NE\_RCOND, **e** is overwritten by the  $(n - 1)$  subdiagonal elements of the unit lower bidiagonal matrix  $L$  from the  $LDL^T$  factorization of  $A$ . (**e** can also be regarded as the superdiagonal of the unit upper bidiagonal factor  $U$  from the  $U^T DU$  factorization of  $A$ .)

6: **b**[*dim*] – double *Input/Output*

**Note:** the dimension, *dim*, of the array **b** must be at least

$$\begin{aligned} &\max(1, \mathbf{pdb} \times \mathbf{nrhs}) \text{ when } \mathbf{order} = \text{Nag\_ColMajor}; \\ &\max(1, n \times \mathbf{pdb}) \text{ when } \mathbf{order} = \text{Nag\_RowMajor}. \end{aligned}$$

The  $(i, j)$ th element of the matrix  $B$  is stored in

$$\begin{aligned} &\mathbf{b}[(j - 1) \times \mathbf{pdb} + i - 1] \text{ when } \mathbf{order} = \text{Nag\_ColMajor}; \\ &\mathbf{b}[(i - 1) \times \mathbf{pdb} + j - 1] \text{ when } \mathbf{order} = \text{Nag\_RowMajor}. \end{aligned}$$

*On entry:* the  $n$  by  $r$  matrix of right-hand sides  $B$ .

*On exit:* if **fail.code** = NE\_NOERROR or NE\_RCOND, the  $n$  by  $r$  solution matrix  $X$ .

7: **pdb** – Integer *Input*

*On entry:* the stride separating row or column elements (depending on the value of **order**) in the array **b**.

*Constraints:*

$$\begin{aligned} &\text{if } \mathbf{order} = \text{Nag\_ColMajor}, \mathbf{pdb} \geq \max(1, n); \\ &\text{if } \mathbf{order} = \text{Nag\_RowMajor}, \mathbf{pdb} \geq \max(1, \mathbf{nrhs}). \end{aligned}$$

8: **rcond** – double \* *Output*

*On exit:* if **fail.code** = NE\_NOERROR or NE\_RCOND, an estimate of the reciprocal of the condition number of the matrix  $A$ , computed as  $\mathbf{rcond} = 1 / (\|A\|_1 \|A^{-1}\|_1)$ .

9: **errbnd** – double \* *Output*

*On exit:* if **fail.code** = NE\_NOERROR or NE\_RCOND, an estimate of the forward error bound for a computed solution  $\hat{x}$ , such that  $\|\hat{x} - x\|_1 / \|x\|_1 \leq \mathbf{errbnd}$ , where  $\hat{x}$  is a column of the computed solution returned in the array **b** and  $x$  is the corresponding column of the exact solution  $X$ . If **rcond** is less than *machine precision*, then **errbnd** is returned as unity.

10: **fail** – NagError \* *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_ALLOC\_FAIL

Dynamic memory allocation failed.

**NE\_BAD\_PARAM**

On entry, argument  $\langle value \rangle$  had an illegal value.

**NE\_INT**

On entry,  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{n} \geq 0$ .

On entry,  $\mathbf{nrhs} = \langle value \rangle$ .

Constraint:  $\mathbf{nrhs} \geq 0$ .

On entry,  $\mathbf{pdb} = \langle value \rangle$ .

Constraint:  $\mathbf{pdb} > 0$ .

**NE\_INT\_2**

On entry,  $\mathbf{pdb} = \langle value \rangle$  and  $\mathbf{n} = \langle value \rangle$ .

Constraint:  $\mathbf{pdb} \geq \max(1, \mathbf{n})$ .

**NE\_INTERNAL\_ERROR**

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

**NE\_POS\_DEF**

The principal minor of order  $\langle value \rangle$  of the matrix  $A$  is not positive definite. The factorization has not been completed and the solution could not be computed.

**NE\_RCOND**

A solution has been computed, but  $\mathbf{rcond}$  is less than *machine precision* so that the matrix  $A$  is numerically singular.

**7 Accuracy**

The computed solution for a single right-hand side,  $\hat{x}$ , satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and  $\epsilon$  is the *machine precision*. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A) \frac{\|E\|_1}{\|A\|_1},$$

where  $\kappa(A) = \|A^{-1}\|_1 \|A\|_1$ , the condition number of  $A$  with respect to the solution of the linear equations. `nag_real_sym_posdef_tridiag_lin_solve` (f04bgc) uses the approximation  $\|E\|_1 = \epsilon \|A\|_1$  to estimate **errbnd**. See Section 4.4 of Anderson *et al.* (1999) for further details.

**8 Parallelism and Performance**

`nag_real_sym_posdef_tridiag_lin_solve` (f04bgc) is not threaded by NAG in any implementation.

`nag_real_sym_posdef_tridiag_lin_solve` (f04bgc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

## 9 Further Comments

The total number of floating-point operations required to solve the equations  $AX = B$  is proportional to  $nr$ . The condition number estimation requires  $O(n)$  floating-point operations.

See Section 15.3 of Higham (2002) for further details on computing the condition number of tridiagonal matrices.

The complex analogue of `nag_real_sym_posdef_tridiag_lin_solve` (f04bgc) is `nag_herm_posdef_tridiag_lin_solve` (f04cgc).

## 10 Example

This example solves the equations

$$AX = B,$$

where  $A$  is the symmetric positive definite tridiagonal matrix

$$A = \begin{pmatrix} 4.0 & -2.0 & 0 & 0 & 0 \\ -2.0 & 10.0 & -6.0 & 0 & 0 \\ 0 & -6.0 & 29.0 & 15.0 & 0 \\ 0 & 0 & 15.0 & 25.0 & 8.0 \\ 0 & 0 & 0 & 8.0 & 5.0 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 6.0 & 10.0 \\ 9.0 & 4.0 \\ 2.0 & 9.0 \\ 14.0 & 65.0 \\ 7.0 & 23.0 \end{pmatrix}.$$

An estimate of the condition number of  $A$  and an approximate error bound for the computed solutions are also printed.

### 10.1 Program Text

```

/* nag_real_sym_posdef_tridiag_lin_solve (f04bgc) Example Program.
 *
 * Copyright 2004 Numerical Algorithms Group.
 *
 * Mark 8, 2004.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf04.h>
#include <nagx04.h>

int main(void)
{
    /* Scalars */
    double      errbnd, rcond;
    Integer     exit_status, i, j, n, nrhs, pdb;

    /* Arrays */
    double      *b, *d = 0, *e = 0;

    /* Nag Types */
    NagError    fail;
    Nag_OrderType order;

#ifdef NAG_COLUMN_MAJOR
#define B(I, J)      b[(J-1)*pdb + I - 1]
    order = Nag_ColMajor;
#else
#define B(I, J)      b[(I-1)*pdb + J - 1]
    order = Nag_RowMajor;
#endif
#define b_ref(a_1, a_2) b[(a_2)*8 + a_1 - 9]

    exit_status = 0;
    INIT_FAIL(fail);

```

```

printf("nag_real_sym_posdef_tridiag_lin_solve (f04bgc) Example "
      "Program Results\n\n");

/* Skip heading in data file */
scanf("%*[\n] ");
scanf("%ld%ld%*[\n] ", &n, &nrhs);
if (n > 0 && nrhs > 0)
{
    /* Allocate memory */
    if (!(b = NAG_ALLOC(n*nrhs, double)) ||
        !(d = NAG_ALLOC(n, double)) ||
        !(e = NAG_ALLOC(n-1, double)))
    {
        printf("Allocation failure\n");
        exit_status = -1;
        goto END;
    }
#ifdef NAG_COLUMN_MAJOR
    pdb = n;
#else
    pdb = nrhs;
#endif
}
else
{
    printf("%s\n", "n and/or nrhs too small");
    exit_status = 1;
    return exit_status;
}

/* Read A from data file */
for (i = 1; i <= n; ++i)
{
    scanf("%lf", &d[i-1]);
}
scanf("%*[\n] ");

for (i = 1; i <= n-1; ++i)
{
    scanf("%lf", &e[i-1]);
}
scanf("%*[\n] ");

/* Read B from data file */
for (i = 1; i <= n; ++i)
{
    for (j = 1; j <= nrhs; ++j)
    {
        scanf("%lf", &B(i, j));
    }
}
scanf("%*[\n] ");

/* Solve the equations AX = B for X */
/* nag_real_sym_posdef_tridiag_lin_solve (f04bgc).
 * Computes the solution and error-bound to a real symmetric
 * positive-definite tridiagonal system of linear equations
 */
nag_real_sym_posdef_tridiag_lin_solve(order, n, nrhs, d, e, b, pdb, &rcond,
                                     &errbnd, &fail);
if (fail.code == NE_NOERROR)
{
    /* Print solution, estimate of condition number and approximate */
    /* error bound */

    /* nag_gen_real_mat_print (x04cac).
     * Print real general matrix (easy-to-use)
     */
    fflush(stdout);
    nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
                          nrhs, b, pdb, "Solution", 0, &fail);
}

```

```

if (fail.code != NE_NOERROR)
{
    printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}
printf("\n%s\n%6s%10.1e\n", "Estimate of condition number", "",
    1.0/rcond);
printf("\n\n");
printf("%s\n%6s",
    "Estimate of error bound for computed solutions",
    "");
printf("%10.1e\n\n", errbnd);
}
else if (fail.code == NE_RCOND)
{
    /* Matrix A is numerically singular. Print estimate of */
    /* reciprocal of condition number and solution */
    printf("\n");
    printf("%s\n%6s%10.1e\n",
        "Estimate of reciprocal of condition number", "", rcond);
    printf("\n\n");

    /* nag_gen_real_mat_print (x04cac), see above. */
    fflush(stdout);
    nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n,
        nrhs, b, pdb, "Solution", 0, &fail);
    if (fail.code != NE_NOERROR)
    {
        printf("Error from nag_gen_real_mat_print (x04cac).\n%s\n",
            fail.message);
        exit_status = 1;
        goto END;
    }
}
else if (fail.code == NE_POS_DEF)
{
    printf("%s%3ld%s\n\n", "The leading minor of order ",
        fail.errnum, " is not positive definite");
}
else
{
    printf(
        "Error from nag_real_sym_posdef_tridiag_lin_solve (f04bgc).\n%s\n",
        fail.message);
    exit_status = 1;
    goto END;
}
END:
NAG_FREE(b);
NAG_FREE(d);
NAG_FREE(e);
return exit_status;
}

```

## 10.2 Program Data

nag\_real\_sym\_posdef\_tridiag\_lin\_solve (f04bgc) Example Program Data

```

5      2                               :Values of N and NRHS

4.0  10.0  29.0  25.0   5.0 :End of diagonal D
-2.0 -6.0  15.0   8.0      :End of sub-diagonal E

6.0  10.0
9.0   4.0
2.0   9.0
14.0 65.0
7.0  23.0                               :End of matrix B

```

### **10.3 Program Results**

nag\_real\_sym\_posdef\_tridiag\_lin\_solve (f04bgc) Example Program Results

Solution

	1	2
1	2.5000	2.0000
2	2.0000	-1.0000
3	1.0000	-3.0000
4	-1.0000	6.0000
5	3.0000	-5.0000

Estimate of condition number  
1.1e+02

Estimate of error bound for computed solutions  
1.2e-14

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