NAG Library Function Document

nag_hermitian_eigenvalues (f02awc)

1 Purpose

nag_hermitian_eigenvalues (f02awc) calculates all the eigenvalues of a complex Hermitian matrix.

2 Specification

```
#include <nag.h>
#include <nagf02.h>
```

```
void nag_hermitian_eigenvalues (Integer n, Complex a[], Integer tda,
     double r[], NagError *fail)
```

3 Description

The complex Hermitian matrix A is first reduced to a real tridiagonal matrix by $n-2$ unitary transformations, and a subsequent diagonal transformation. The eigenvalues are then derived using the QL algorithm, an adaptation of the QR algorithm.

4 References

Wilkinson J H and Reinsch C (1971) *Handbook for Automatic Computation II, Linear Algebra* Springer–Verlag

5 Arguments

1: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 1$.

2: **a**[**n** × **tda**] – Complex *Input/Output*

Note: the (i,j)th element of the matrix A is stored in $\mathbf{a}[(i-1) \times \mathbf{tda} + j - 1]$.

On entry: the elements of the lower triangle of the n by n complex Hermitian matrix A. Elements of the array above the diagonal need not be set.

On exit: **a** is overwritten.

3: **tda** – Integer *Input*

On entry: the stride separating matrix column elements in the array **a**.

Constraint: $\mathbf{tda} \geq \mathbf{n}$.

4: **r**[**n**] – double *Output*

On exit: the eigenvalues in ascending order.

5: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_2_INT_ARG_LT

On entry, **tda** $= \langle value \rangle$ while **n** $= \langle value \rangle$. These arguments must satisfy **tda** $\geq$ **n**.

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_INT_ARG_LT

On entry, **n** $= \langle value \rangle$.
Constraint: **n** ≥ 1.

NE_TOO_MANY_ITERATIONS

More than $\langle value \rangle$ iterations are required to isolate all the eigenvalues.

7 Accuracy

For a detailed error analysis see page 235 of Wilkinson and Reinsch (1971).

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken by nag_hermitian_eigenvalues (f02awc) is approximately proportional to n^3.

10 Example

To calculate all the eigenvalues of the complex Hermitian matrix:

$$\begin{pmatrix} 0.50 & 0.00 & 1.84+1.38i & 2.08-1.56i \\ 0.00 & 0.50 & 1.12+0.84i & -0.56+0.42i \\ 1.84-1.38i & 1.12-0.84i & 0.50 & 0.00 \\ 2.08+1.56i & -0.56-0.42i & 0.00 & 0.50 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_hermitian_eigenvalues (f02awc) Example Program.
 *
 * Copyright 1991 Numerical Algorithms Group.
 *
 * Mark 2, 1991.
 * Mark 8 revised, 2004.
 */

#include <nag.h>
#include <stdio.h>
#include <nag_stdlib.h>
#include <nagf02.h>

#define A(I, J) a[(I) *tda + J]
int main(void)
{
  Complex  *a = 0;
  Integer  exit_status = 0, i, j, n, tda;
  NagError fail;
  double   *r = 0;

  INIT_FAIL(fail);
  printf(
          "nag_hermitian_eigenvalues (f02awc) Example Program Results\n");
  scanf("%*[^\n]"); /* Skip heading in data file */
  scanf("%ld", &n);
  if (n >= 1)
    {
      if (!(r = NAG_ALLOC(n, double)) ||
          !(a = NAG_ALLOC(n*n, Complex)))
        {
          printf("Allocation failure\n");
          exit_status = -1;
          goto END;
        }
      tda = n;
    }
  else
    {
      printf("Invalid n.\n");
      exit_status = 1;
      return exit_status;
    }
  for (i = 0; i < n; i++)
    for (j = 0; j < n; j++)
      scanf(" ( %lf,  %lf ) ", &A(i, j).re, &A(i, j).im);
  /* nag_hermitian_eigenvalues (f02awc).
   * All eigenvalues of complex Hermitian matrix
   */
  nag_hermitian_eigenvalues(n, a, tda, r, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_hermitian_eigenvalues (f02awc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }
  printf("Eigenvalues\n");
  for (i = 0; i < n; i++)
    printf("%9.4f", r[i]);
  printf("\n");
 END:
  NAG_FREE(r);
  NAG_FREE(a);
  return exit_status;
}
```

10.2 Program Data

```
nag_hermitian_eigenvalues (f02awc) Example Program Data
  4
(0.50, 0.00) ( 0.00, 0.00) (1.84,1.38) ( 2.08,-1.56)
(0.00, 0.00) ( 0.50, 0.00) (1.12,0.84) (-0.56, 0.42)
(1.84,-1.38) ( 1.12,-0.84) (0.50,0.00) ( 0.00, 0.00)
(2.08, 1.56) (-0.56,-0.42) (0.00,0.00) ( 0.50, 0.00)
```

10.3 Program Results

```
nag_hermitian_eigenvalues (f02awc) Example Program Results
Eigenvalues
  -3.0000  -1.0000   2.0000   4.0000
```