

NAG Library Function Document

nag_matop_real_gen_matrix_log (f01ejc)

1 Purpose

nag_matop_real_gen_matrix_log (f01ejc) computes the principal matrix logarithm, $\log(A)$, of a real n by n matrix A , with no eigenvalues on the closed negative real line.

2 Specification

```
#include <nag.h>
#include <nagf01.h>

void nag_matop_real_gen_matrix_log (Nag_OrderType order, Integer n,
    double a[], Integer pda, double *imnorm, NagError *fail)
```

3 Description

Any nonsingular matrix A has infinitely many logarithms. For a matrix with no eigenvalues on the closed negative real line, the principal logarithm is the unique logarithm whose spectrum lies in the strip $\{z : -\pi < \text{Im}(z) < \pi\}$.

$\log(A)$ is computed using the inverse scaling and squaring algorithm for the matrix logarithm described in Al-Mohy and Higham (2011), adapted to real matrices by Al-Mohy *et al.* (2012).

4 References

Al-Mohy A H and Higham N J (2011) Improved inverse scaling and squaring algorithms for the matrix logarithm *SIAM J. Sci. Comput.* **34(4)** C152–C169

Al-Mohy A H, Higham N J and Relton S D (2012) Computing the Fréchet derivative of the matrix logarithm and estimating the condition number *MIMS EPrint* **2012.72**

Higham N J (2008) *Functions of Matrices: Theory and Computation* SIAM, Philadelphia, PA, USA

5 Arguments

1: **order** – Nag_OrderType *Input*

On entry: the **order** argument specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = Nag_RowMajor. See Section 3.2.1.3 in the Essential Introduction for a more detailed explanation of the use of this argument.

Constraint: **order** = Nag_RowMajor or Nag_ColMajor.

2: **n** – Integer *Input*

On entry: n , the order of the matrix A .

Constraint: $n \geq 0$.

3: **a**[*dim*] – double *Input/Output*

Note: the dimension, *dim*, of the array **a** must be at least **pda** × **n**.

The (i, j)th element of the matrix A is stored in

$$\begin{aligned} & \mathbf{a}[(j-1) \times \mathbf{pda} + i - 1] \text{ when } \mathbf{order} = \text{Nag_ColMajor}; \\ & \mathbf{a}[(i-1) \times \mathbf{pda} + j - 1] \text{ when } \mathbf{order} = \text{Nag_RowMajor}. \end{aligned}$$

On entry: the n by n matrix A .

On exit: the n by n principal matrix logarithm, $\log(A)$.

4: **pda** – Integer

Input

On entry: the stride separating row or column elements (depending on the value of **order**) in the array **a**.

Constraint: **pda** \geq **n**.

5: **imnorm** – double *

Output

On exit: if the function has given a reliable answer then **imnorm** = 0.0. If **imnorm** differs from 0.0 by more than unit roundoff (as returned by nag_machine_precision (X02AJC)) then the computed matrix logarithm is unreliable.

6: **fail** – NagError *

Input/Output

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Allocation of memory failed.

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_EIGENVALUES

A was found to have eigenvalues on the negative real line. The principal logarithm is not defined in this case. nag_matop_complex_gen_matrix_log (f01fjc) can be used to find a complex non-principal logarithm.

NE_INT

On entry, **n** = $\langle value \rangle$.

Constraint: **n** \geq 0.

NE_INT_2

On entry, **pda** = $\langle value \rangle$ and **n** = $\langle value \rangle$.

Constraint: **pda** \geq **n**.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_SINGULAR

A is singular so the logarithm cannot be computed.

NW_SOME_PRECISION_LOSS

$\log(A)$ has been computed using an IEEE double precision Padé approximant, although the arithmetic precision is higher than IEEE double precision.

7 Accuracy

For a normal matrix A (for which $A^T A = A A^T$), the Schur decomposition is diagonal and the algorithm reduces to evaluating the logarithm of the eigenvalues of A and then constructing $\log(A)$ using the Schur vectors. This should give a very accurate result. In general, however, no error bounds are available for the algorithm. See Al-Mohy and Higham (2011) and Section 9.4 of Higham (2008) for details and further discussion.

The sensitivity of the computation of $\log(A)$ is worst when A has an eigenvalue of very small modulus or has a complex conjugate pair of eigenvalues lying close to the negative real axis.

If estimates of the condition number of the matrix logarithm are required then `nag_matop_real_gen_matrix_cond_log` (f01jjc) should be used.

8 Parallelism and Performance

`nag_matop_real_gen_matrix_log` (f01ejc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_matop_real_gen_matrix_log` (f01ejc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The cost of the algorithm is $O(n^3)$ floating-point operations (see Al-Mohy and Higham (2011)). The double allocatable memory required is approximately $3 \times n^2$.

If the Fréchet derivative of the matrix logarithm is required then `nag_matop_real_gen_matrix_frcht_log` (f01jkc) should be used.

`nag_matop_complex_gen_matrix_log` (f01fjc) can be used to find the principal logarithm of a complex matrix. It can also be used to return a complex, non-principal logarithm if a real matrix has no principal logarithm due to the presence of negative eigenvalues.

10 Example

This example finds the principal matrix logarithm of the matrix

$$A = \begin{pmatrix} 3 & -3 & 1 & 1 \\ 2 & 1 & -2 & 1 \\ 1 & 1 & 3 & -1 \\ 2 & 0 & 2 & 0 \end{pmatrix}.$$

10.1 Program Text

```

/* nag_matop_real_gen_matrix_log (f01ejc) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <math.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf01.h>
#include <nagx04.h>

int main(void)
{

```

```

/* Scalars */
Integer      exit_status = 0;
Integer      i, j, n, pda;
double       imnorm;

/* Arrays */
double       *a = 0;

/* Nag Types */
Nag_OrderType order;
NagError     fail;

INIT_FAIL(fail);

#ifdef NAG_COLUMN_MAJOR
#define A(I, J) a[(J-1)*pda + I-1]
  order = Nag_ColMajor;
#else
#define A(I, J) a[(I-1)*pda + J-1]
  order = Nag_RowMajor;
#endif

/* Output preamble */
printf("nag_matop_real_gen_matrix_log (f01ejc) ");
printf("Example Program Results\n\n");
fflush(stdout);

/* Skip heading in data file */
scanf("%*[\n]");

/* Read in the problem size */
scanf("%ld%*[\n]", &n);

pda = n;

if (!(a = NAG_ALLOC(pda*n, double))) {
  printf("Allocation failure\n");
  exit_status = -1;
  goto END;
}

/* Read in the matrix a from data file */
for (i = 1; i <= n; i++)
  for (j = 1; j <= n; j++) scanf("%lf", &A(i, j));
scanf("%*[\n]");

/* Find log(A) using
 * nag_matop_real_gen_matrix_log (f01ejc)
 * Real matrix logarithm
 */
nag_matop_real_gen_matrix_log(order, n, a, pda, &imnorm, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_matop_real_gen_matrix_log (f01ejc)\n%s\n",
        fail.message);
  exit_status = 1;
  goto END;
}

/* Print solution using
 * nag_gen_real_mat_print (x04cac)
 * Print real general matrix (easy-to-use)
 */
nag_gen_real_mat_print(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, n,
                      a, pda, "log(A)", NULL, &fail);
if (fail.code != NE_NOERROR)
{
  printf("Error from nag_gen_real_mat_print (x04cac)\n%s\n", fail.message);
  exit_status = 2;
}

```

```

        goto END;
    }

END:
    NAG_FREE(a);
    return exit_status;
}

```

10.2 Program Data

```

nag_matop_real_gen_matrix_log (f01ejc) Example Program Data
4                               :Value of n
3.0   -3.0    1.0    1.0
2.0    1.0   -2.0    1.0
1.0    1.0    3.0   -1.0
2.0    0.0    2.0    0.0 :End of matrix a

```

10.3 Program Results

```

nag_matop_real_gen_matrix_log (f01ejc) Example Program Results

```

```

log(A)
      1      2      3      4
1    1.1957  -1.2076  -0.5802  1.0872
2    0.8464   1.0133  -0.5985  -0.1641
3    0.4389   0.6701   1.8449  -1.2111
4    1.2792   0.6177   2.1448  -1.9743

```
