NAG Library Function Document

nag_1d_aitken_interp (e01aac)

1 Purpose

nag_1d_aitken_interp (e01aac) interpolates a function of one variable at a given point x from a table of function values y_i evaluated at equidistant or non-equidistant points x_i, for $i = 1, 2, \ldots, n + 1$, using Aitken's technique of successive linear interpolations.

2 Specification

```
#include <nag.h>
#include <nage01.h>
void nag_1d_aitken_interp (Integer n, double a[], double b[], double c[],
     double x, NagError *fail)
```

3 Description

nag_1d_aitken_interp (e01aac) interpolates a function of one variable at a given point x from a table of values x_i and y_i, for $i = 1, 2, \ldots, n + 1$ using Aitken's method (see Fröberg (1970)). The intermediate values of linear interpolations are stored to enable an estimate of the accuracy of the results to be made.

4 References

Fröberg C E (1970) *Introduction to Numerical Analysis* Addison–Wesley

5 Arguments

1: **n** – Integer *Input*

On entry: the number of intervals which are to be used in interpolating the value at x; that is, there are $n + 1$ data points (x_i, y_i).

Constraint: **n** > 0.

2: $\mathbf{a}[\mathbf{n}+\mathbf{1}]$ – double *Input/Output*

On entry: $\mathbf{a}[i-1]$ must contain the x-component of the ith data point, x_i, for $i = 1, 2, \ldots, n + 1$.

On exit: $\mathbf{a}[i-1]$ contains the value $x_i - x$, for $i = 1, 2, \ldots, n + 1$.

3: $\mathbf{b}[\mathbf{n}+\mathbf{1}]$ – double *Input/Output*

On entry: $\mathbf{b}[i-1]$ must contain the y-component (function value) of the ith data point, y_i, for $i = 1, 2, \ldots, n + 1$.

On exit: the contents of **b** are unspecified.

4: $\mathbf{c}[\mathbf{n}\times(\mathbf{n}+\mathbf{1})/\mathbf{2}]$ – double *Output*

On exit:

$\mathbf{c}[0], \ldots, \mathbf{c}[n-1]$ contain the first set of linear interpolations,

$\mathbf{c}[n], \ldots, \mathbf{c}[2\times n-2]$ contain the second set of linear interpolations,

$\mathbf{c}[2n-1], \ldots, \mathbf{c}[3\times n-4]$ contain the third set of linear interpolations,

$\vdots$

$\mathbf{c}[n \times (n+1)/2 - 1]$ contains the interpolated function value at the point x.

5: **x** – double *Input*

On entry: the point x at which the interpolation is required.

6: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_BAD_PARAM

On entry, argument $\langle value \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{n} = \langle value \rangle$.
Constraint: $\mathbf{n} > 0$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

An estimate of the accuracy of the result can be made from a comparison of the final result and the previous interpolates, given in the array **c**. In particular, the first interpolate in the ith set, for $i = 1, 2, \ldots, n$, is the value at x of the polynomial interpolating the first $(i+1)$ data points. It is given in position $(i-1)(2n-i+2)/2$ of the array **c**. Ideally, providing n is large enough, this set of n interpolates should exhibit convergence to the final value, the difference between one interpolate and the next settling down to a roughly constant magnitude (but with varying sign). This magnitude indicates the size of the error (any subsequent increase meaning that the value of n is too high). Better convergence will be obtained if the data points are supplied, not in their natural order, but ordered so that the first i data points give good coverage of the neighbourhood of x, for all i. To this end, the following ordering is recommended as widely suitable: first the point nearest to x, then the nearest point on the opposite side of x, followed by the remaining points in increasing order of their distance from x, that is of $|x_r - x|$. With this modification the Aitken method will generally perform better than the related method of Neville, which is often given in the literature as superior to that of Aitken.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The computation time for interpolation at any point x is proportional to $n \times (n+1)/2$.

10 Example

This example interpolates at $x = 0.28$ the function value of a curve defined by the points

$$\begin{pmatrix} x_i & -1.00 & -0.50 & 0.00 & 0.50 & 1.00 & 1.50 \\ y_i & 0.00 & -0.53 & -1.00 & -0.46 & 2.00 & 11.09 \end{pmatrix}.$$

10.1 Program Text

```
/* nag_1d_aitken_interp (e01aac) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nage01.h>

int main(void)
{
  /* Scalars */
  Integer exit_status = 0;
  Integer i, j, k, n;
  double  x;
  NagError fail;
  /* Arrays */
  double  *a = 0, *b = 0, *c = 0;

  INIT_FAIL(fail);

  printf("nag_1d_aitken_interp (e01aac) Example Program Results\n\n");

  /* Skip heading in data file*/
  scanf("%*[^\n] ");
  scanf("%"NAG_IFMT "", &n);
  scanf("%lf", &x);
  scanf("%*[^\n] ");

  /* Allocate memory */
  if (!(a = NAG_ALLOC((n+1), double)) ||
      !(b = NAG_ALLOC((n+1), double)) ||
      !(c = NAG_ALLOC((n*(n+1)/2), double)))
    {
      printf("Allocation failure\n\n");
      exit_status = -1;
      goto END;
    }

  for (i = 0; i <= n; i++)
    scanf("%lf", &a[i]);
  scanf("%*[^\n] ");
  for (i = 0; i <= n; i++)
    scanf("%lf", &b[i]);
  scanf("%*[^\n] ");

  /* nag_1d_aitken_interp (e01aac).
   * Interpolated values, Aitken's technique,
   * unequally spaced data, one variable.
   */
  nag_1d_aitken_interp(n, a, b, c, x, &fail);
  if (fail.code != NE_NOERROR){
    printf("Error from nag_1d_aitken_interp (e01aac).\n%s\n",
           fail.message);
    exit_status = 1;
    goto END;
  }

  printf("Interpolated values\n");
  k = 0;
  for (i = 1; i <= n - 1; i++){
    for (j = k; j <= k + n - i; j++)
      printf("%12.5f", c[j]);
    printf("\n");
    k = j;
  }
```

```
  printf("\nInterpolation point = %12.5f\n", x);
  printf("\nFunction value at interpolation point = %12.5f\n", c[n*(n+1)/2-1]);

 END:
  NAG_FREE(a);
  NAG_FREE(b);
  NAG_FREE(c);

  return exit_status;
}
```

10.2 Program Data

```
nag_1d_aitken_interp (e01aac) Example Program Data
  5     0.28
    -1.00   -0.50    0.00    0.50    1.00    1.50
     0.00   -0.53   -1.00   -0.46    2.00   11.09
```

10.3 Program Results

```
nag_1d_aitken_interp (e01aac) Example Program Results

Interpolated values
    -1.35680    -1.28000    -0.39253     1.28000     5.67808
    -1.23699    -0.60467     0.01434     1.38680
    -0.88289    -0.88662    -0.74722
    -0.88125    -0.91274

Interpolation point =      0.28000

Function value at interpolation point =     -0.83591
```