NAG Library Function Document

nag_quad_md_numth_coeff_2prime (d01gzc)

1 Purpose

nag_quad_md_numth_coeff_2prime (d01gzc) calculates the optimal coefficients for use by nag_quad_md_numth_vec (d01gdc), when the number of points is the product of two primes.

2 Specification

```
#include <nag.h>
#include <nagd01.h>
void nag_quad_md_numth_coeff_2prime (Integer ndim, Integer np1, Integer np2,
     double vk[], NagError *fail)
```

3 Description

Korobov (1963) gives a procedure for calculating optimal coefficients for p-point integration over the n-cube $[0,1]^n$, when the number of points is

$$p = p_1 p_2 \tag{1}$$

where p_1 and p_2 are distinct prime numbers.

The advantage of this procedure is that if p_1 is chosen to be the nearest prime integer to p_2^2, then the number of elementary operations required to compute the rule is of the order of $p^{4/3}$ which grows less rapidly than the number of operations required by nag_quad_md_numth_coeff_prime (d01gyc). The associated error is likely to be larger although it may be the only practical alternative for high values of p.

4 References

Korobov N M (1963) *Number Theoretic Methods in Approximate Analysis* Fizmatgiz, Moscow

5 Arguments

1: **ndim** – Integer *Input*

On entry: n, the number of dimensions of the integral.

Constraint: **ndim** ≥ 1.

2: **np1** – Integer *Input*

On entry: the larger prime factor p_1 of the number of points in the integration rule.

Constraint: **np1** must be a prime number ≥ 5.

3: **np2** – Integer *Input*

On entry: the smaller prime factor p_2 of the number of points in the integration rule. For maximum efficiency, p_2^2 should be close to p_1.

Constraint: **np2** must be a prime number such that **np1** $>$ **np2** ≥ 2.

4: **vk**[**ndim**] – double *Output*

On exit: the n optimal coefficients.

5: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ACCURACY

The ***machine precision*** is insufficient to perform the computation exactly. Try reducing **np1** or **np2**: **np1** = ⟨*value*⟩ and **np2** = ⟨*value*⟩.

NE_BAD_PARAM

On entry, argument ⟨*value*⟩ had an illegal value.

NE_INT

On entry, **ndim** = ⟨*value*⟩.
Constraint: **ndim** ≥ 1.

On entry, **np1** = ⟨*value*⟩.
Constraint: **np1** must be a prime number.

On entry, **np1** = ⟨*value*⟩.
Constraint: **np1** ≥ 5.

On entry, **np2** = ⟨*value*⟩.
Constraint: **np2** must be a prime number.

On entry, **np2** = ⟨*value*⟩.
Constraint: **np2** ≥ 2.

NE_INT_2

On entry, **np1** × **np2** exceeds largest machine integer. **np1** = ⟨*value*⟩ and **np2** = ⟨*value*⟩.

On entry, **np1** = ⟨*value*⟩ and **np2** = ⟨*value*⟩.
Constraint: **np1** $>$ **np2**.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

The optimal coefficients are returned as exact integers (though stored in a double array).

8 Parallelism and Performance

Not applicable.

9 Further Comments

The time taken by nag_quad_md_numth_coeff_2prime (d01gzc) grows at least as fast as $(p_1 p_2)^{4/3}$. (See Section 3.)

10 Example

This example calculates the Korobov optimal coefficients where the number of dimensons is 4 and the number of points is the product of the two prime numbers, 89 and 11.

10.1 Program Text

```
/* nag_quad_md_numth_coeff_2prime (d01gzc) Example Program.
 *
 * Copyright 2011, Numerical Algorithms Group.
 *
 * Mark 23, 2011.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagd01.h>

int main(void)
{
  Integer exit_status = 0;
  Integer i, ndim, np1, np2;
  double *vk = 0;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_quad_md_numth_coeff_2prime (d01gzc) Example Program Results\n");
  /* Skip heading in data file */
  scanf("%*[^\n] ");
  scanf("%ld", &ndim);
  scanf("%ld %ld%*[^\n] ", &np1, &np2);

  if (!(vk = NAG_ALLOC(ndim, double)))
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* nag_quad_md_numth_coeff_2prime (d01gzc).
   * Korobov optimal coefficients for use in d01gdc,
   * when number of points is product of two primes.
   */
  nag_quad_md_numth_coeff_2prime(ndim, np1, np2, vk, &fail);
  if (fail.code != NE_NOERROR)
    {
      printf("Error from nag_quad_md_numth_coeff_2prime (d01gzc).\n%s\n",
              fail.message);
      exit_status = 1;
      goto END;
    }

  printf("\nndim = %3ld np1 = %6ld np2 = %6ld\n",
         ndim, np1, np2);
  printf("\nCoefficients =");
  for (i = 0; i < ndim; i++)
    printf("%4.0f ", vk[i]);
  printf("\n");

 END:
  NAG_FREE(vk);

  return exit_status;
}
```

10.2 Program Data

None.

10.3 Program Results

```
nag_quad_md_numth_coeff_2prime (d01gzc) Example Program Results

ndim =   4 np1 =     89 np2 =     11

Coefficients =   1  102  614  951
```