

NAG Library Function Document

nag_sum_fft_hermitian_2d (c06pwc)

1 Purpose

nag_sum_fft_hermitian_2d (c06pwc) computes the two-dimensional inverse discrete Fourier transform of a bivariate Hermitian sequence of complex data values.

2 Specification

```
#include <nag.h>
#include <nagc06.h>

void nag_sum_fft_hermitian_2d (Integer m, Integer n, const Complex y[],
    double x[], NagError *fail)
```

3 Description

nag_sum_fft_hermitian_2d (c06pwc) computes the two-dimensional inverse discrete Fourier transform of a bivariate Hermitian sequence of complex data values $z_{j_1 j_2}$, for $j_1 = 0, 1, \dots, m-1$ and $j_2 = 0, 1, \dots, n-1$.

The discrete Fourier transform is here defined by

$$\hat{x}_{k_1 k_2} = \frac{1}{\sqrt{mn}} \sum_{j_1=0}^{m-1} \sum_{j_2=0}^{n-1} z_{j_1 j_2} \times \exp\left(2\pi i \left(\frac{j_1 k_1}{m} + \frac{j_2 k_2}{n}\right)\right),$$

where $k_1 = 0, 1, \dots, m-1$ and $k_2 = 0, 1, \dots, n-1$. (Note the scale factor of $\frac{1}{\sqrt{mn}}$ in this definition.)

Because the input data satisfies conjugate symmetry (i.e., $z_{k_1 k_2}$ is the complex conjugate of $z_{(m-k_1)k_2}$, the transformed values $\hat{x}_{k_1 k_2}$ are real.

A call of nag_sum_fft_real_2d (c06pvc) followed by a call of nag_sum_fft_hermitian_2d (c06pwc) will restore the original data.

This function performs multiple one-dimensional discrete Fourier transforms by the fast Fourier transform (FFT) algorithm in Brigham (1974) and Temperton (1983).

4 References

Brigham E O (1974) *The Fast Fourier Transform* Prentice–Hall

Temperton C (1983) Fast mixed-radix real Fourier transforms *J. Comput. Phys.* **52** 340–350

5 Arguments

- | | | |
|----|--|--------------|
| 1: | m – Integer
<i>On entry:</i> m , the first dimension of the transform.
<i>Constraint:</i> $m \geq 1$. | <i>Input</i> |
| 2: | n – Integer
<i>On entry:</i> n , the second dimension of the transform.
<i>Constraint:</i> $n \geq 1$. | <i>Input</i> |

- 3: $\mathbf{y}[(\mathbf{m}/2 + 1) \times \mathbf{n}] - \text{const Complex}$ *Input*
On entry: the Hermitian sequence of complex input dataset z , where $z_{j_1 j_2}$ is stored in $\mathbf{y}[j_2 \times (\mathbf{m}/2 + 1) + j_1]$, for $j_1 = 0, 1, \dots, \mathbf{m}/2$ and $j_2 = 0, 1, \dots, \mathbf{n} - 1$.
- 4: $\mathbf{x}[\mathbf{m} \times \mathbf{n}] - \text{double}$ *Output*
On exit: the real output dataset \hat{x} , where $\hat{x}_{k_1 k_2}$ is stored in $\mathbf{x}[k_2 \times \mathbf{m} + k_1]$, for $k_1 = 0, 1, \dots, \mathbf{m} - 1$ and $k_2 = 0, 1, \dots, \mathbf{n} - 1$.
- 5: **fail** – NagError * *Input/Output*
 The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_ALLOC_FAIL

Dynamic memory allocation failed.

NE_BAD_PARAM

On entry, argument $\langle \text{value} \rangle$ had an illegal value.

NE_INT

On entry, $\mathbf{m} = \langle \text{value} \rangle$.

Constraint: $\mathbf{m} \geq 1$.

On entry, $\mathbf{n} = \langle \text{value} \rangle$.

Constraint: $\mathbf{n} \geq 1$.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

7 Accuracy

Some indication of accuracy can be obtained by performing a forward transform using `nag_sum_fft_real_2d` (c06pvc) and a backward transform using `nag_sum_fft_hermitian_2d` (c06pwc), and comparing the results with the original sequence (in exact arithmetic they would be identical).

8 Parallelism and Performance

`nag_sum_fft_hermitian_2d` (c06pwc) is threaded by NAG for parallel execution in multithreaded implementations of the NAG Library.

`nag_sum_fft_hermitian_2d` (c06pwc) makes calls to BLAS and/or LAPACK routines, which may be threaded within the vendor library used by this implementation. Consult the documentation for the vendor library for further information.

Please consult the Users' Note for your implementation for any additional implementation-specific information.

9 Further Comments

The time taken by `nag_sum_fft_hermitian_2d` (c06pwc) is approximately proportional to $mn \log(mn)$, but also depends on the factors of m and n . `nag_sum_fft_hermitian_2d` (c06pwc) is fastest if the only prime factors of m and n are 2, 3 and 5, and is particularly slow if m or n is a large prime, or has large prime factors.

Workspace is internally allocated by `nag_sum_fft_hermitian_2d` (c06pwc). The total size of these arrays is approximately proportional to mn .

10 Example

See Section 10 in `nag_sum_fft_real_2d` (c06pvc).
