NAG Library Function Document

nag_quartic_roots (c02alc)

1 Purpose

nag_quartic_roots (c02alc) determines the roots of a quartic equation with real coefficients.

2 Specification

```
#include <nag.h>
#include <nagc02.h>
void nag_quartic_roots (double e, double a, double b, double c, double d,
     double zeror[], double zeroi[], double errest[], NagError *fail)
```

3 Description

nag_quartic_roots (c02alc) attempts to find the roots of the quartic equation

$$ez^4 + az^3 + bz^2 + cz + d = 0,$$

where e, a, b, c and d are real coefficients with $e \neq 0$. The roots are located by finding the eigenvalues of the associated 4 by 4 (upper Hessenberg) companion matrix H given by

$$H = \begin{pmatrix} -d/e & & & \\ -c/e & & & \\ -b/e & & & \\ -a/e & & & \end{pmatrix}.$$

Further details can be found in Section 9.

To obtain the roots of a cubic equation, nag_cubic_roots (c02akc) can be used.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Arguments

1: **e** – double *Input*

On entry: e, the coefficient of z^4.

Constraint: **e** $\neq 0.0$.

2: **a** – double *Input*

On entry: a, the coefficient of z^3.

3: **b** – double *Input*

On entry: b, the coefficient of z^2.

4: **c** – double *Input*

On entry: c, the coefficient of z.

5: **d** – double *Input*

On entry: d, the constant coefficient.

6: **zeror**[**4**] – double *Output*
7: **zeroi**[**4**] – double *Output*

On exit: **zeror**$[i-1]$ and **zeroi**$[i-1]$ contain the real and imaginary parts, respectively, of the ith root.

8: **errest**[**4**] – double *Output*

On exit: **errest**$[i-1]$ contains an approximate error estimate for the ith root.

9: **fail** – NagError * *Input/Output*

The NAG error argument (see Section 3.6 in the Essential Introduction).

6 Error Indicators and Warnings

NE_C02_NOT_CONV

The iterative procedure used to determine the eigenvalues has failed to converge.

NE_C02_OVERFLOW

The companion matrix H cannot be formed without overflow.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please contact NAG for assistance.

NE_REAL

On entry, **e** $= 0.0$.
Constraint: **e** $\neq 0.0$.

7 Accuracy

If **fail** = NE_NOERROR on exit, then the ith computed root should have approximately $|\log_{10}(\textbf{errest}[i-1])|$ correct significant digits.

8 Parallelism and Performance

Not applicable.

9 Further Comments

The method used by the function consists of the following steps, which are performed by functions from LAPACK.

(a) Form matrix H.

(b) Apply a diagonal similarity transformation to H (to give H').

(c) Calculate the eigenvalues and Schur factorization of H'.

(d) Calculate the left and right eigenvectors of H'.

(e) Estimate reciprocal condition numbers for all the eigenvalues of H'.

(f) Calculate approximate error estimates for all the eigenvalues of H' (using the 1-norm).

10 Example

To find the roots of the quartic equation

$$z^4 + 2z^3 + 6z^2 - 8z - 40 = 0.$$

10.1 Program Text

```
/* nag_quartic_roots (c02alc) Example Program.
 *
 * Copyright 2000 Numerical Algorithms Group.
 *
 * NAG C Library
 *
 * Mark 6, 2000.
 * Mark 7, revised, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagc02.h>

int main(void)
{
  double   a, b, c, d, e;
  double   *errest = 0, *zeroi = 0, *zeror = 0;
  Integer  i;
  Integer  exit_status = 0;
  NagError fail;

  INIT_FAIL(fail);

  printf("nag_quartic_roots (c02alc) Example Program Results\n\n");
  if
  (
    !(errest = NAG_ALLOC(4, double)) ||
    !(zeroi = NAG_ALLOC(4, double)) ||
    !(zeror = NAG_ALLOC(4, double))
  )
    {
      printf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Skip heading in data file */
  scanf("%*[^\n]");
  scanf("%lf %lf %lf %lf %lf", &e, &a, &b, &c, &d);

  /* nag_quartic_roots (c02alc).
   * Zeros of a real quartic polynomial with real coefficients
   */
  nag_quartic_roots(e, a, b, c, d, zeror, zeroi, errest, &fail);
  if (fail.code == NE_NOERROR)
    {
      printf(" Roots of quartic equation          Error estimates\n");
      printf(
             "                                    (machine-dependent)\n\n");
      for (i = 0; i <= 3; ++i)
        {
          printf("%s %10.5f %10.5f%s        %g\n", " z =",
                  zeror[i], zeroi[i], "*i", errest[i]);
        }
    }
  else
    {
      printf("Error from nag_quartic_roots (c02alc).\n%s\n",
              fail.message);
      exit_status = 1;
```

```
      goto END;
    }
 END:
  NAG_FREE(errest);
  NAG_FREE(zeroi);
  NAG_FREE(zeror);
  return exit_status;
}
```

10.2 Program Data

```
nag_quartic_roots (c02alc) Example Program Data
 1.0   2.0   6.0  -8.0  -40.0 : Values of e, a, b, c and d
```

10.3 Program Results

```
nag_quartic_roots (c02alc) Example Program Results

 Roots of quartic equation          Error estimates
                                   (machine-dependent)

 z =    2.00000    0.00000*i        3.39305e-15
 z =   -2.00000    0.00000*i        5.29913e-15
 z =   -1.00000    3.00000*i        4.54822e-15
 z =   -1.00000   -3.00000*i        4.54822e-15
```